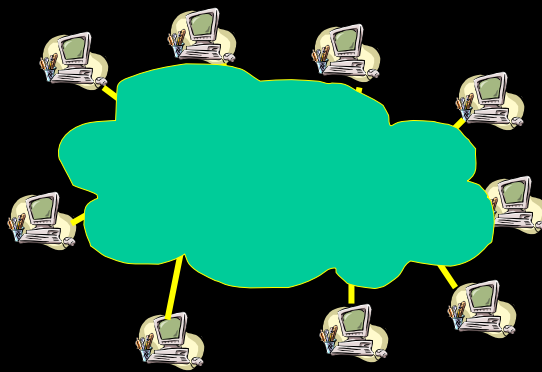


Algorithms, Networks and Games

Christos H. Papadimitriou
UC Berkeley



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The Internet

- Huge, growing, open, end-to-end
- The platform of the www
- Built by 20,000 selfish agents
- *The first computational artefact that must be studied by the scientific method:*
- Observations, experiments, falsifiable theories, *specialized applied math:*
- Game theory, graph theory, algorithms

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Outline

- “The Price of Anarchy”
- Mechanism Design
- Algorithms for Equilibria

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Collaborators:

Costas Daskalakis, Joan Feigenbaum,
Paul Goldberg, Dick Karp, Elias Koutsoupias,
Aranyak Mehta, Milena Mihail, Amin Saberi,
Rahul Sami, Scott Shenker

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Games help us understand
rational behavior
in competitive situations

matching pennies

1,-1	-1,1
-1,1	1,-1

chicken

0,0	0,1
1,0	-1,-1

prisoner's dilemma

3,3	0,4
4,0	1,1

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Nash equilibrium

- Definition: all-around best response
- Problem: May not exist
- *Randomized* Nash equilibrium always exists [Nash 1951]
- Problem: There may be many

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Exploring the multiplicity: “The price of anarchy”

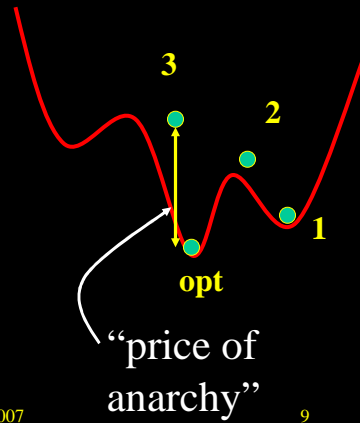
$$\text{p. of } \textcircled{A} = \frac{\text{cost of worst Nash equilibrium}}{\text{“socially optimum” cost}}$$

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“The Third Compromise”

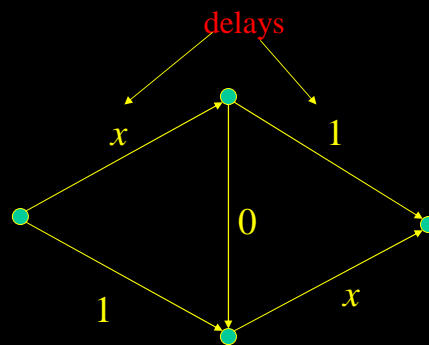
1. Problems are hard, so approximate
2. Crucial information is scarce, so we need to decide on-line, or in isolation
3. distributed optimizers may have conflicting objectives...



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Selfishness can hurt you!



Social optimum: 1.5

Anarchical equilibrium: 2

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Worst case?

Price of anarchy = “2” (4/3 for linear delays)

[Roughgarden and Tardos, 2000,
Roughgarden 2002]

The price of the Internet architecture?

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Mechanism design (or *inverse* game theory)

- agents have utilities – but these utilities are known *only to them*
- designed game (mechanism) has socially optimum outcome *no matter what the agents' true objectives were*
- main trick: bribe agents to tell the truth

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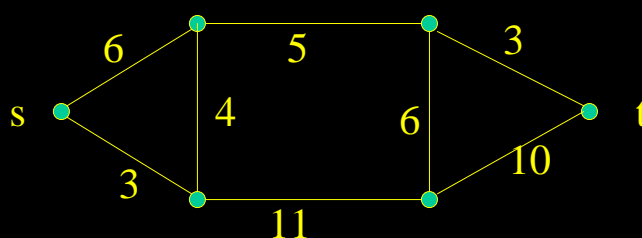
e.g., Vickrey auction

- sealed-highest-bid auction encourages gaming and speculation
- Vickrey auction: Highest bidder wins, *pays second-highest bid*
- Participants are incentivized to tell the truth: *Truthful mechanism*

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e.g., shortest path auction

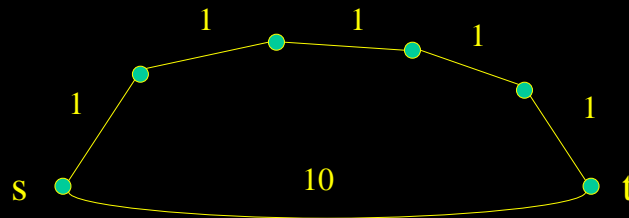


pay e its declared cost $c(e)$,
plus a bonus equal to $\text{dist}(s,t)|_{c(e)=\infty} - \text{dist}(s,t)$

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Problem:



Theorem [Elkind, Sahai, Steiglitz, 03]: This is inherent in any truthful mechanism.

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But...

- ...in the Internet (the graph of autonomous systems) VCG overcharge would be only about 30% on the average [FPSS 2002]
- Could this be the manifestation of rational behavior at network creation?

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Naaah...

- **Theorem** [with Mihail and Saberi, 2003]:
In a random graph with average degree d ,
the expected VCG overcharge is constant
(*conjectured*: $\sim 1/d$)

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btw: How does *traffic* grow?
(*or: why is the Internet backbone overprovisioned?*)

- Trees: n^2
- Expanders (and most degree-balanced sparse graphs): $\sim n$
- The Internet?

Theorem (with Mihail and Saberi, 2003):
“Scale-free sparse graph models” are almost
certainly expanders

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The complexity of equilibria MFAQ

Why?

- Equilibria are notions of *rationality*, aspiring models of *behavior*
- Efficient computability is an important modeling prerequisite
- “If your laptop can’t find it, neither can the market”

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Nash equilibrium in **P**?

- Nash’s proof is non-constructive
- It relies on Brouwer’s fixpoint theorem
- *That* is intractable [HPV88, P95]
- Technical term: *PPAD-complete*
- But is this dependency on Brouwer inherent?
- Or is finding Nash equilibria easier?

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Recent breakthrough:

- Finding a Nash equilibrium is intractable
PPAD-complete [GP05, DGP05]
- Even for 4 players
- Basic idea is *arithmetization*: Games that add
and multiply!

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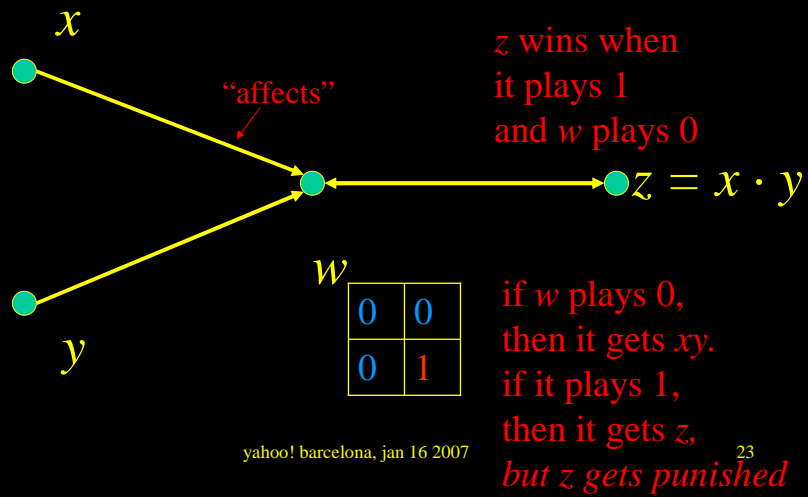
*“multiplication
is the name of the game
and each generation
plays the same”*

Bobby Darren, 1961

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The multiplication game



So....



Brouwer \equiv Nash

Open problems

- Conjecture: 3 players also PPAD-complete
- *Proved! November 05 [CD, CP]*
- Conjecture: 2 players in P
- *Wrong!?!? December 05
PPAD-complete [Chen&Deng]*

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Game over!

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Important remaining problem

Approximate Nash equilibrium: Defecting players can only gain ε

- $\varepsilon = .75$ [KPS06]
- $\varepsilon = .5$ [DMP06]
- $\varepsilon = .32\dots$ [DMP07]
- Open: any $\varepsilon > 0$ in time $O(n^{1/\varepsilon})$